



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 60

Interest Rate Futures – 2

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STIRF APPLICATIONS



T BILL FUTURES LOCK IN FORWARD RATE

- Assume that you take a long position in a T-bill future @ $F_0(0,T,N)$ that matures for delivery at $t=T$ and entails delivery of a T bill of face value F . This T-Bill matures for repayment at $t=N$.
- This means that you receive the T bill at $t=T$ and pay the price F_0 at the same time ($t=T$). **However, this price F_0 is fixed at $t=0$.**

- You will receive the **proceeds (face value F)** of the T bill at $t=N$.
- Hence, your cash flows at $t=T$ ($-F_0$) and $t=N$ ($+F$) are fully determined at $t=0$ (subject to MTM).
- Thus, you have effectively locked the forward rate for the period $t=T$ and $t=N$.

LOCKING A FORWARD RATE



LOCKING A SPOT RATE



	t=0		t=T	
	RECEIVE	GIVE	RECEIVE	GIVE
BUY N DAY T BILL SPOT	T BILL	$(-)P(N)$		
SELL T BILL FUTURE (T,N)			$F_0(0,T,N)$	$(-)T \text{ BILL}$
TOTAL	T BILL	$(-)P(N)$	$F_0(0,T,N)$	T BILL

HEDGING WITH IRFs: BASIC PRINCIPLES

- Let us assume that today is $t=0$ and you take a long position i.e. buy a 90 day T-Bill future that has T days to delivery.
- Let at any point in the interval $0 < t < T$, the interest rate make a **shift downwards**.
- As a result, the price of the underlying 90 day T-Bill will go up.
- Since, you have a long position under the futures contract i.e. you have a right to buy the 90 day T-Bill (underlying) at the predetermined price (which is lower), your leg of the future will **command a positive value** i.e. you will earn a profit.



- Hence, if your primary investment is such that you incur a **loss due to a fall in interest rates**, then by taking this **long position in T-Bill futures** you will be able to compensate for the loss on the exposure by the profit that you make on the T-Bill future.



HEDGING AGAINST INTEREST RATE RISK: EXAMPLES

- **When interest rates decrease long futures yield a profit.**
- You hold \$10 m 91-day T-bills which you will sell in one month's time because you need to pay your creditors.
- If market interest rates increase, the value of your investment portfolio of T-Bills will decline.
- Hence, your hedge must yield a profit when interest rates increase.
- **SHORT**



- You will receive \$10m in 6 months time which you want to invest in T-bills for 90 days.
- If interest rates decline in the intervening 6 months, the price of T-bills will increase and you will get less face value of T-bills for your investment. Hence your return will decrease.
- Thus, your hedge should generate a profit when interest rates decline.
- LONG

- You hope to issue \$10m of 180-day commercial paper in 3 month time (i.e. to borrow money).
- If interest rates increase in the intervening 3 months, the price of CPs will decrease and you will get less cash inflow by issuing the CPs. Hence your borrowings become more costly.
- Thus, your hedge should generate a profit when interest rates increase.
- **SHORT**

HEDGING WITH T-BILLS

HEDGING AN EXISTING PORTFOLIO

$$V_{0,s} = F_s \left(1 - d_{s0} \frac{N}{360} \right); \quad V_{T,s} = F_s \left[1 - d_{sT} \left(\frac{N - T}{360} \right) \right]$$

$$dV_s = V_{T,s} - V_{0,s} = F_s \left[d_{s0} \frac{N}{360} - d_{sT} \left(\frac{N - T}{360} \right) \right] \quad \xrightarrow{\hspace{10em}}$$

0
T
N

$$= F_s \left[d_{s0} \left(\frac{N - T}{360} \right) - d_{sT} \left(\frac{N - T}{360} \right) + d_{s0} \left(\frac{T}{360} \right) \right] = -F_s \Delta d_s \left(\frac{N - T}{360} \right) + F_s d_{s0} \left(\frac{T}{360} \right)$$

But $F_s d_{s0} \left(\frac{T}{360} \right)$ is the change in value even if yield is unchanged.

Hence, change in value of exposure due to yield change

$$dS = -F_s \Delta d_s \left(\frac{N - T}{360} \right)$$

For each futures contract, $P_{0,f} = F_f^* (1 - 0.25d_{f0})$; $P_{T,f} = F_f^* [1 - 0.25d_{fT}]$

F_f^* = Face value per futures contract

Change in value of N_f futures $dF = -0.25N_f F_f^* \Delta d_f$

For minimum variance hedge: $N_f = \frac{(N - T)F_s}{360 \times 0.25 \times F_f^*} \beta_{\Delta d_s, \Delta d_f}$

OBSERVATIONS

- The entire stochasticity of yields is captured by the regression coefficient β (hedge ratio).
- If we assume **parallel shifts of the spot yield curve**, then $\beta_{\Delta y_s, \Delta y_f}$ being the slope of the regression line between $\Delta y_s, \Delta y_f$ is unity because interest rates for all maturities change with the same amount and so the change in Δy_f will be accompanied by an equal change in Δy_s and vice versa.



IMPORTANT

$$N_f = \frac{(N - T)F_s}{360 \times 0.25 \times F_f^*} \beta_{\Delta d_s, \Delta d_f}$$

- The value of the futures position is proportional to the remaining lifespan of the hedged T-bills (N-T).
- This is so because any yield shift will impact the value of the T bill over this span.
- Hence, greater the lifespan, greater will be the price change of the T bill for a given yield shift.

EXAMPLE 1

- Today is $t=0$. XYZ Ltd is holding 364 day T-Bills (FV of \$10m) with a maturity of $t=12$ months. Its investment horizon is upto $t=2$ months. The company decides to hedge through 3 month T-Bill futures with delivery at $t=5$ months. The hedge will be lifted at $t=2$ months by the closing of the futures contracts.

- The **IMM index quotes** at $t=0$ and $t=2$ months of the 3 month T-Bill futures turn out to be 88.00 and 86.00 respectively. **Spot discount yields** on T-bills at $t=0$ and $t=2$ months are 9% and 11% respectively. Work out the extent to which the hedge has operated successfully. Assume yield shifts to be parallel.



NO OF CONTRACTS (SHORT)	$N_f = \frac{(N - T)F_s}{360 \times 0.25 \times F_f^*} \beta_{\Delta d_s, \Delta d_f}$	
VALUE OF PORTFOLIO	10000000	
LIFE OF T BILLS	12 MONTHS	
PERIOD OF HEDGING	2 MONTHS	
RESIDUAL LIFE	10 MONTHS	
FV OF EACH FUTURES	1000000	
NO OF FUTURES	33	$(N-T)*F(s)/3F^*(f)$ months

PERFORMANCE OF HEDGE	0	2
FUTURES QUOTES	88	86
DISCOUNT YIELD	0.12	0.14
FUTURES PRICE $P = F(1 - 0.25y) = 1(1 - 0.25 \times 0.12) = 0.97$	970000	965000
PROFIT PER CONTRACT		5000
PROFIT ON FUTURES POSITION		165000
CASH POSITION		
DISCOUNT YIELD	0.09	0.11
REMAINING MATURITY	1	0.833333333
PRICE	910000	908333.3333
PRICE AT ORIGINAL YIELD		925000
LOSS DUE TO CHANGE IN YIELD		16666.66667
TOTAL LOSS DUE TO CHANGE IN YIELD ON 10 BILLS		166666.6667
HENCE CHANGE IN VALUE OF HEDGED POSITION		-1,666.67

HEDGING A FUTURE INVESTMENT

Let the amount to be invested V_s at time T

Then, we have $F_s = V_s \left(1 + r_{s0} \frac{N - T}{360} \right)$

Let the actual interest rate at time T be r_{sT}

Actual investment $F_s^* = V_s \left(1 + r_{sT} \frac{N - T}{360} \right)$

Change in cash position $F_s^* - F_s = V_s (r_{sT} - r_{s0}) \left(\frac{N - T}{360} \right) = V_s \Delta r_s \left(\frac{N - T}{360} \right)$



Hence, change in value of exposure due to yield change

$$dF_s = V_s \left(\frac{N - T}{360} \right) \Delta r_s$$

For each futures, $P_{0,f} = F_f^* (1 - 0.25d_{f0})$; $P_{T,f} = F_f^* [1 - 0.25d_{fT}]$

Change in value of N_f futures $dF = -0.25N_f F_f^* \Delta d_f$

$$N_f = \frac{(N - T) V_s}{360 * 0.25 F_f^*} \beta_{\Delta r_s, \Delta d_f}$$

EXAMPLE 2

- It is $t=0$ and the treasurer of company X expects to receive \$1m in $t=3$ months, and wishes to invest this in a 6-month T-bill until $t=9$ months. The treasurer fears a fall in interest rates, which would imply that his investment beginning in $t=3$ months will earn less interest. The treasurer buys (**goes long**) 3 month T-bill futures today ($t=0$) with maturity in $t=4$ months.

The IMM index quotes at $t=0$ and $t=3$ of the T-Bill futures turn out to be 89.2 and 90.3 respectively. Spot investment interest rates at $t=0$ and $t=3$ months are 11% and 9.6% respectively. Work out the extent to which the hedge has operated successfully. Assume yield shifts to be parallel.



NO OF CONTRACTS	$N_f =$	$\frac{(N - T) V_s}{360 * 0.25 F_f^*}$	$\beta_{\Delta r_s, \Delta d_f}$
FACE VALUE OF INVESTMENT (V(s))	1000000		
MATURITY OF INVESTMENT FROM NOW (N)	9 MONTHS		
TIMING OF INVESTMENT (T)	3 MONTHS		
RESIDUAL LIFE (N-T)	6 MONTHS		
FV OF EACH BILL FUTURE (F*)	1000000		
NO OF FUTURES	2	$(N-T)*V(s)/3F^*$	months

PERFORMANCE OF HEDGE	0	3
FUTURES IMM QUOTES	89.2	90.3
FUTURES DISCOUNT YIELD	0.108	0.097
FUTURES PRICE	973000	975750
PROFIT PER CONTRACT (LONG)		2750
PROFIT ON FUTURES POSITION		5500
CASH POSITION		
INTEREST RATE	0.11	0.096
MATURITY	0.5	0.5
INTEREST INCOME	55000	48000
LOSS DUE TO CHANGE IN INTT RATE		-7000
PV OF THE LOSS		-6686.406299
NET LOSS		-1186.406299